Section 4.3 Trigonometry Extended: The Circular Functions

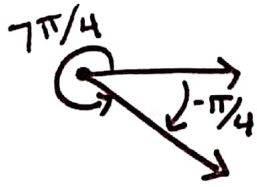
Two angles are coterminal when they have the same initial and terminal sides. To find an angle that is coterminal to a given angle θ add or subtract 2π (one revolution). A given angle θ has infinitely many coterminal angles. -OR- (add/subtract 360° if θ in degrees)

Coterminal Angles

If θ is the measure of an angle in radians, then all angles measuring $\theta + 2n\pi$, where n is an integer, are coterminal with θ.

Example 1: For each problem draw the angles/ coterminal relationship.

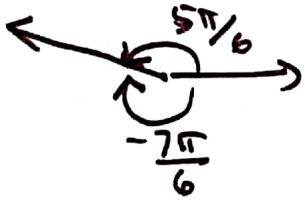
a. For the positive angle $\frac{7\pi}{4}$, subtract 2π to obtain a coterminal angle.



$\frac{7\pi}{4} - 2\pi$

$\frac{7\pi}{4} - \frac{8\pi}{4} = \frac{-\pi}{4}$

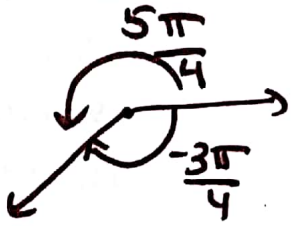
b. For the positive angle $\frac{5\pi}{6}$, subtract 2π to obtain a coterminal angle.



$\frac{5\pi}{6} - \frac{12\pi}{6} = \frac{-7\pi}{6}$

c. For the negative angle $-\frac{3\pi}{4}$, add 2π to find a coterminal angle.

$-\frac{3\pi}{4} + \frac{8\pi}{4} = \frac{5\pi}{4}$ (+2π)

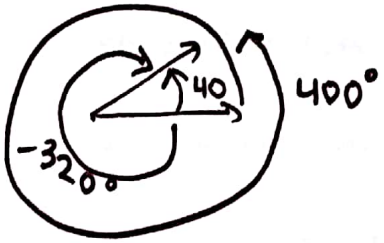


Example 2: Find and draw a positive and negative coterminal angle with the given angle

a. 40°

+360 = 400°

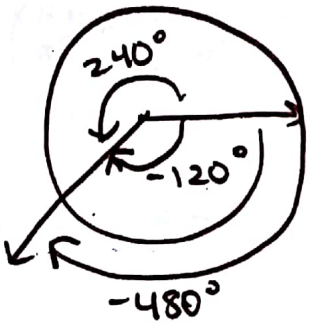
-360 = -320°



b. -120°

+360 = 240°

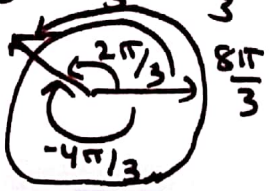
-360 = -480°



c. $\frac{2\pi}{3}$ radians

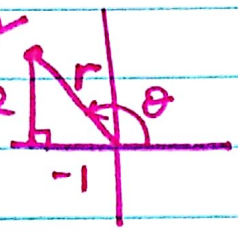
+2π = $\frac{8\pi}{3}$

-2π = $-\frac{4\pi}{3}$



Notes 4.3 (classwork odds)

3) QII



$(-1)^2 + 2^2 = r^2$

$1 + 4 = r^2$

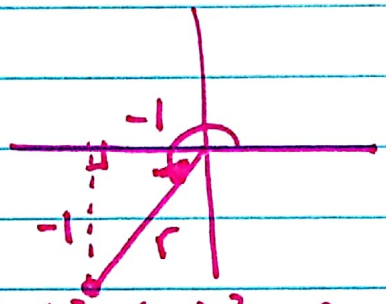
$\sqrt{5} = r$

$\sin\theta = \frac{2}{\sqrt{5}}$ $\quad \csc\theta = \frac{\sqrt{5}}{2}$

$\cos\theta = -\frac{1}{\sqrt{5}}$ $\quad \sec\theta = -\sqrt{5}$

$\tan\theta = -2$ $\quad \cot\theta = -\frac{1}{2}$

5) QIII



$(-1)^2 + (-1)^2 = r^2$

$1 + 1 = r^2$

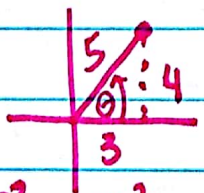
$\sqrt{2} = r$

$\sin\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad \csc\theta = -\sqrt{2}$

$\cos\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad \sec\theta = -\sqrt{2}$

$\tan\theta = 1$ $\quad \cot\theta = 1$

7) P(3,4)



$3^2 + 4^2 = r^2$

$9 + 16 = r^2$

$25 = r^2$

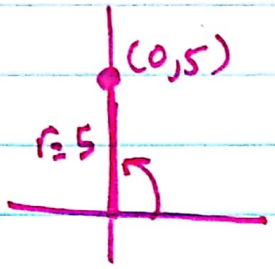
$5 = r$

$\sin\theta = \frac{4}{5}$ $\quad \csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\quad \sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

9) $P(0,5)$

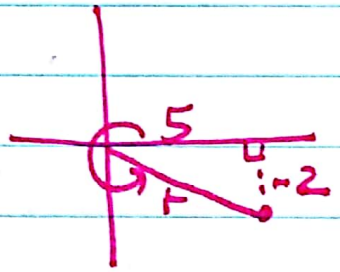


$\sin\theta = \frac{5}{5} = 1$ $\quad \csc\theta = 1$

$\cos\theta = \frac{0}{5} = 0$ $\quad \sec\theta = \text{undefined}$

$\tan\theta = \text{undefined}$ $\quad \frac{5}{0} \rightarrow$ $\quad \cot\theta = 0$ $\quad \frac{0}{5} \rightarrow$

11) $P(5,-2)$



$(-2)^2 + 5^2 = r^2$

$4 + 25 = r^2$

$\sqrt{29} = r$

$\sin\theta = \frac{-2}{\sqrt{29}}$ $\quad \csc\theta = -\frac{\sqrt{29}}{2}$

$\cos\theta = \frac{5}{\sqrt{29}}$ $\quad \sec\theta = \frac{\sqrt{29}}{5}$

$\tan\theta = -\frac{2}{5}$ $\quad \cot\theta = -\frac{5}{2}$

13) $\left(0, \frac{\pi}{2}\right)$ QI

a) sint +

b) cost +

c) tant +

15) $\left(\pi, \frac{3\pi}{2}\right)$ QIII

a) sint = −

b) cost −

c) tant +

Use unit circle

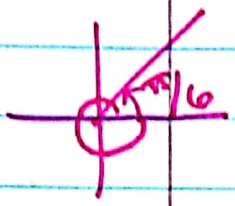
25) $\cos 120° = \boxed{-\frac{1}{2}}$

27) $\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{1/2}$

$= \boxed{2}$

29) $\sin\frac{13\pi}{6} = \sin\frac{\pi}{6} = \boxed{\frac{1}{2}}$

$\frac{12\pi}{6} + \frac{\pi}{6}$



31) $\tan\left(\frac{-15\pi}{4}\right)$

$-\frac{8\pi}{4} + \frac{-7\pi}{4}$

QI

$= \tan\left(-\frac{7\pi}{4}\right)$

$= \tan\frac{\pi}{4} = \boxed{1}$

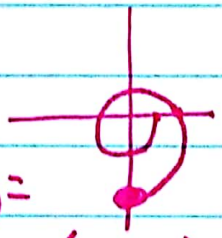
33) $\cos\frac{23\pi}{6} = \cos\frac{11\pi}{6} = \boxed{\frac{\sqrt{3}}{2}}$

$\frac{12\pi}{6} + \frac{11\pi}{6}$

35) $\sin\frac{11\pi}{3} = \sin\frac{5\pi}{3}$

$\frac{6\pi}{3} + \frac{5\pi}{3} = \boxed{-\frac{\sqrt{3}}{2}}$

37) $-450°$



$\sin(-450°) =$

$\sin(-90°) = (0,-1)$

$\boxed{-1}$

$\cos(-450°) =$

$\cos(-90°) =$

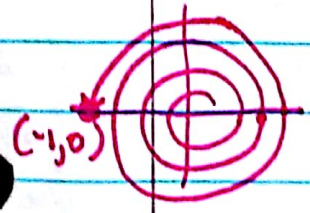
$\boxed{0}$

$\tan(-450)$

$\tan(-90)$

$\boxed{\text{undefined}}$

39) 7π



$\sin(7\pi) =$

$\sin\pi = \boxed{0}$

$\cos(7\pi)$

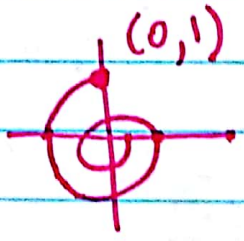
$\cos\pi = \boxed{-1}$

$\tan(7\pi)$

$\tan\pi = \boxed{0}$

41) $-\frac{7\pi}{2}$

$-4\pi/2 + -3\pi/2$



$\sin\left(-\frac{7\pi}{2}\right) = \sin\left(-\frac{3\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

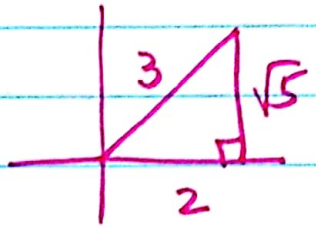
$\cos\left(-\frac{7\pi}{2}\right) = \cos\left(-\frac{3\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

$\tan\left(-\frac{7\pi}{2}\right) = \tan\left(-\frac{3\pi}{2}\right) = \tan\frac{\pi}{2} = \text{UND.}$

(−,+)	(+,+)
(−,−)	(+,−)

④

43) $\cos\theta = \frac{2}{3}$ $\cot\theta > 0$ QI



$2^2 + y^2 = 3^2$

$4 + y^2 = 9$

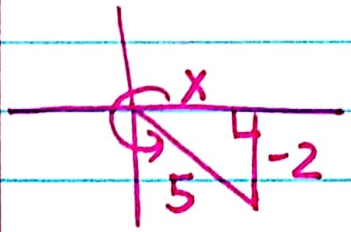
$y^2 = 5$

$y = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sqrt{5}}{2}$

45) $\sin\theta = -\frac{2}{5}$ $\cos\theta > 0$ QIV



$\tan\theta = \frac{-2}{\sqrt{21}}$

$x^2 + (-2)^2 = 5^2$

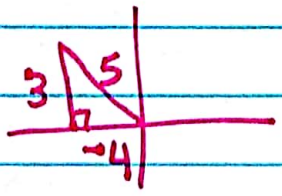
$x^2 + 4 = 25$

$x^2 = 21$

$x = \sqrt{21}$

$\sec\theta = \frac{5}{\sqrt{21}}$

47) $\cot\theta = -\frac{4}{3}$ $\cos\theta < 0$ QII



$3^2 + (-4)^2 = r^2$

$r = 5$

$\sec\theta = \frac{5}{-4} = -\frac{5}{4}$

$\csc\theta = \frac{5}{3}$